The minimum Reynolds number for a turbulent boundary layer and the selection of a transition device

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SUMMARY

In the case of turbulent flow in a pipe there is a lower experimental number to the Reynolds limit for which fully developed turbulent flow occurs. From the similarity and close agreement of the curves showing the coefficient of skin friction c_f as a function of the Reynolds number R_θ (based on the momentum thickness θ) for the circular pipe and flat plate, it is suggested that there should be a lower limit to R_θ for fully developed turbulent flow on a flat plate. Rather limited experimental data confirm this and place the lower limit at $R_\theta = 320$. The choice and size of transition device is examined in relation to this minimum R_θ and an approximate theory leads to a 'wire' Reynolds number in fair agreement with experience.

1. INTRODUCTION

In the literature it is common to find a curve depicting the turbulent skin friction coefficient c_f of a flat plate as a function of the Reynolds number R_x based on the distance x from the leading edge. It is assumed that transition occurs at the leading edge and that the boundary layer has zero thickness there, i.e. $R_\theta = 0$. This is an assumption that has a certain analytical convenience, but it has dangers for the experimenter and the engineer. The former is tempted to induce transition at the leading edge by means of a wire or other device without giving much thought as to what are the real conditions just downstream of it, whilst the latter makes estimates of drag for this assumed state which may be considerably in error at low values of R_x.

In this connection the recent paper by Dutton (1956) on the turbulent boundary layer of a flat plate brings out clearly the marked effect of the size and type of transition device on the distribution of c_f and R_θ considered as functions of R_x. However, Dutton finds c_f to be an approximately unique function of R_θ and independent of the transition device used. This is consistent with the assumption of universal 'inner' and 'outer' laws for the velocity distribution which has been the basis of important papers by Landweber (1953) and by Coles (1954) and will also be used in this paper.

In this note, with the aid of pipe flow results and the results obtained by Dutton and earlier investigators on a flat plate, it will be suggested that there is a lower experimental limit to the value of R_θ for the turbulent boundary layer of a flat plate as for turbulent flow in a pipe. Theoretical support for this suggestion is given and the functions of a transition device in relation to this idea of a lower limit are then considered.

2. Flow in circular pipes

Figure 1 shows Nikuradse's (1932, 1933) results for $\tau_w/(\frac{1}{2}\rho U_m)$ as a function of $R = U_m d/\nu$ and the roughness ratio $d/2\epsilon$, where U_m is the mean velocity in the pipe, d is the pipe diameter, τ_w is the skin friction, and ϵ is the roughness height. As R is decreased, the various curves tend to that for the smooth pipe (turbulent flow) with the exception of the one for the largest roughness $d/2\epsilon = 15$. At a value of $\log_{10} R$ of about 3·65, all the curves, which have now merged into one, travel down a common

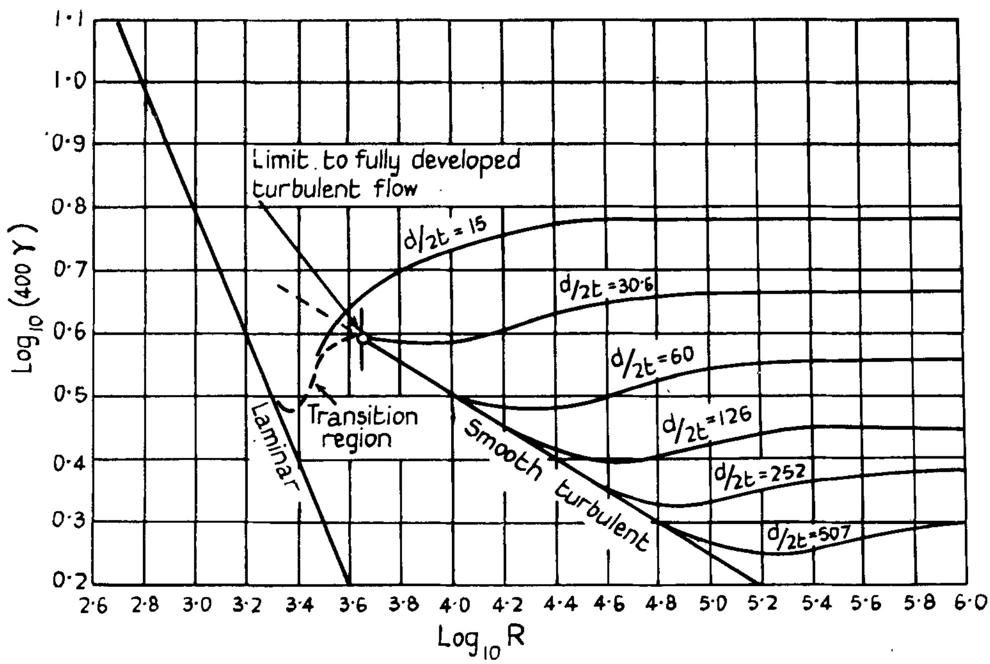

Figure 1. Resistance of rough pipes (after Nikuradse).

transition curve which joins the curve for laminar flow at $\log_{10} R \doteqdot 3{\cdot}32$. This appears to be the maximum value of R for laminar flow with very disturbed entry conditions and, even with comparatively large closely packed roughness, the skin friction has the laminar flow value at Reynolds numbers below this critical value.

Now the significant point from the standpoint of the present note is that it is impossible to obtain results which would extend the smooth pipe (turbulent flow) curve to values of $\log_{10} R < 3{\cdot}65$ or $R < 4{\cdot}47 \times 10^3$, and this is the lowest Reynolds number for fully developed turbulent flow in a pipe.

Defining the momentum thickness for axisymmetric flow by

$$\theta = \int_0^{d/2} \frac{u}{U_1}\left(1 - \frac{u}{U_1}\right)\left(1 - \frac{2r}{d}\right) dr,$$

where U_1 is the velocity of the pipe centre and u is the velocity at radius r, it is possible to obtain $2\theta/d$ as a function of the Reynolds number R when the velocity distribution is known. This has been done by Ross (1952)

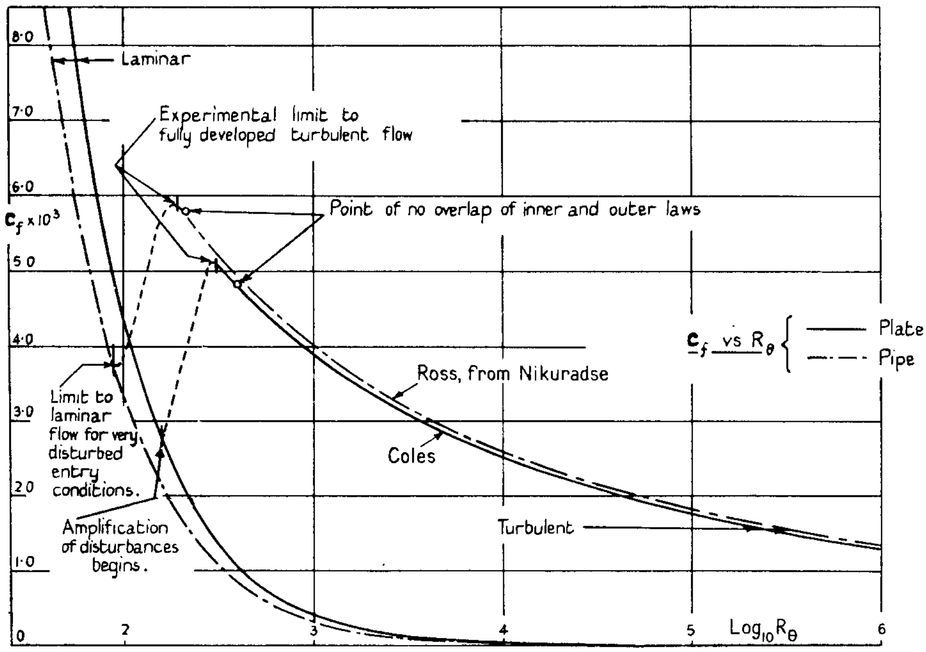

Figure 2. Skin friction coefficient against momentum thickness Reynolds number.

in a re-analysis of Nikuradse's results to yield $R_\theta = U_1 \theta/\nu$ as a function of R, and $c_f = \tau_w/(\frac{1}{2}\rho U^2)$ as a function of R_θ, for turbulent flow. The result has been expressed in the form

$$c_f^{-1/2} = 3{\cdot}8 \log_{10} R_\theta + 4{\cdot}4. \tag{1}$$

This is shown in figure 2 together with the laminar flow relation

$$c_f = \tfrac{1}{3} R_\theta. \tag{2}$$

The minimum Reynolds number for which equation (1) is applicable is $R = 4{\cdot}47 \times 10^3$, and the corresponding value of R_θ is 193. The maximum value of R_θ for laminar flow with very disturbed entry conditions is 87·5,

3. The flow along a flat plate

Of a large number of papers dealing with the problem it is proposed in this paper to refer to only three, the paper by Dutton (1956) already referred to, and the papers by Landweber (1953) and Coles (1954), both of which review much of the existing work on the subject. In the calculation of the growth of the turbulent boundary and the distribution of skin friction, both Landweber and Coles start from assumed velocity distributions through the boundary layer.

In a finite region next to the surface, the 'inner' law is assumed to apply and the velocity u is given by

$$\frac{u}{u_\tau} = f\left(\frac{u_\tau y}{\nu}\right), \tag{3}$$

where $u_\tau = (\tau_w/\rho)^{1/2}$ and y is the distance from the surface. In the outer region of the boundary layer, the 'outer' law applies in the form

$$\frac{U-u}{u_\tau} = \phi(y/\delta), \tag{4}$$

where U is the velocity at the edge $y = \delta$ of the boundary layer.

The assumption (supported by experiment) that these laws overlap over a finite region leads to the well-known logarithmic forms of the equations (3) and (4) in this region

$$u/u_\tau = A + B\log(u_\tau y/\nu) \tag{5}$$

$$(U-u)/u_\tau = C - B\log(y/\delta). \tag{6}$$

The 'inner' law equation (5) is assumed to be universal for all flows, but the 'outer' law for the plate differs from that for the pipe in the value of the constant C in equation (6). The assumption of the 'inner' and 'outer' laws leads in both the plate and pipe problems to unique relations between c_f and R_θ.

In the case of the flat plate flow, the momentum equation

$$\tfrac{1}{2}c_f = d\theta/dx \tag{7}$$

enables the distribution of c_f and the growth of θ to be found for given starting conditions of the turbulent flow. The relations between c_f and R_θ as obtained by Landweber and Coles differ because of the different values of the constants A, B, C assumed in the analysis depending on the individual assessment of the available experimental results. The relation obtained by Coles is utilized in this note because Dutton's results are close to it, and in figure 2 it is given for comparison with that for turbulent flow in a pipe, equation (1). The relation for laminar flow over a flat plate

$$c_f = 0{\cdot}441 R_\theta^{-1} \tag{8}$$

is also plotted.

The general closeness and similarity of the results for pipe and plate is rather striking and it seems reasonable to expect that, as in the case of the pipe, there is a minimum value of R_θ for turbulent flow along a plate. Examination of the results (from various sources) suggests that the minimum value of R_θ for the turbulent boundary layer of a flat plate is about 320 and Dutton's results, which are discussed later, confirm this.

Now both Landweber and Coles recognized that, in computing the boundary layer development along a plate, some initial starting condition has to be satisfied as there is a constant of integration at disposal. Landweber in fact restricted his solution to large Reynolds numbers R_x, so that the starting condition would have negligible effect and he took the constant to be zero. Coles fixed the constant by supposing that $R_\theta = 0$ when $R_x = 0$ and that the skin friction remains finite. Clearly, the constant of integration must be fixed by the value of R_θ at the transition position and this must equal or exceed the value of 320. The value will be fixed by the extent of laminar flow and the drag of the transition device.

Landweber appears to have been the first to recognize that there is a limit to the application of the 'inner' and 'outer' laws in their logarithmic form, since the amount of overlap decreases as the Reynolds number decreases. His argument is as follows. The 'inner' law equation (3) begins to depart from the logarithmic relation equation (5) as the sub-layer is approached and the value of y $(= y_1)$ when this occurs is often taken as given by

$$u_\tau y_1/\nu \doteqdot 30. \tag{9}$$

The outer law equation (4) has from experimental data a logarithmic behaviour as given by equation (6) up to

$$y_2/\delta \doteqdot 0{\cdot}2. \tag{10}$$

These equations specify the degree of overlap of the two laws. If $y_1 = y$, then there is no overlap, and this occurs, from equations (9) and (10), when $u_\tau \delta/\nu = 150$ or

$$\log_{10}(u_\tau \delta/\nu) = 2{\cdot}176. \tag{11}$$

Pipe flow

We now apply this idea to turbulent flow in a pipe for which Ross (1952) gives

$$\left.\begin{aligned} u/u_\tau &= 5{\cdot}6 + 5{\cdot}6\log_{10}(u_\tau y/\nu) \\ (U-u)/u_\tau &= 0{\cdot}785 - 5{\cdot}6\log(y/\delta) \end{aligned}\right\}, \tag{12}$$

and, eliminating u,

$$U/u_\tau = (2/c_f)^{1/2} = 6{\cdot}385 + 5{\cdot}6\log_{10}(u_\tau \delta/\nu). \tag{13}$$

Inserting the value of $\log_{10}(u_\tau \delta/\nu)$ from equation (11) into equation (13), we find the 'no overlap condition gives $c_f = 0{\cdot}0058$, and from figure 2 $\log_{10} R_\theta = 2{\cdot}305$, giving $R_\theta = 202$. This limit is marked on figure 2 and it is seen that it is very close to the experimental limit for fully developed turbulent flow $R_\theta = 193$, *Plate flow*

Coles's relations for the velocity distribution in the overlap region are

$$\left.\begin{aligned} u/u_\tau &= 5{\cdot}10 + 5{\cdot}75\log_{10}(u_\tau y/\nu) \\ (U-u)/u_\tau &= 2{\cdot}80 - 5{\cdot}75\log_{10}(y/\delta) \end{aligned}\right\}, \tag{14}$$

and eliminating u

$$U/u_\tau = (2/c_f)^{1/2} = 7{\cdot}9 + 5{\cdot}75\log_{10}(u_\tau \delta/\nu). \tag{15}$$

The 'no overlap' condition gives $c_f = 0{\cdot}0048$, and from figure 2 the corresponding value of R_θ is given by $\log_{10} R_\theta = 2{\cdot}59$, i.e. $R_\theta = 3{\cdot}89$. The lowest value of R_θ in the data considered by Coles for which fully developed turbulent flow occurs is 320, which is again close to the R_θ for no overlap of the 'inner' and 'outer' regions.

It is not clear whether much significance should be attached to this measure of agreement found for both pipe and plate, owing to the rather arbitrary method of defining the limits of the logarithmic region by equations (9) and (10). Slightly different choice of constants in these equations would yield significantly different values of c_f and R_θ for absence of overlap. But it is clear that, at values of R_θ of this order, the sub-layer is now an appreciable part of the boundary layer and the viscous stresses are becoming important well away from the wall. Experimentally, it seems that when the overlap region vanishes further reduction of R_θ leads to the velocity distribution assuming more and more nearly the characteristics of laminar flow with a rapid decrease in skin friction to the laminar flow value.

Dutton's results

Dutton's work was carried out at fairly low Reynolds numbers and further evidence of a minimum value of R_θ for turbulent flow over a flat plate can be gleaned from his results.

Figure 3 is a reproduction of figure 7 of Dutton's report and shows R_θ as a function of R_x for two transition devices, circular wires and sandpaper strips. The curve for laminar flow has been included in figure 3 and the increments to R_θ caused by wire drag have been estimated by the theory given in the next section. The estimated total R_θ immediately behind the wires have been plotted on this figure and are made up as shown in table 1.

In discussing these results we note that R_θ for the laminar boundary layer just ahead of the wire is 120 and that this is less than the value of $R_\theta = 162$ at which disturbances are first amplified (based on $R_{\delta^*} = 420$, where R_{δ^*} is the Reynolds number formed by the displacement thickness). It is clear from curve (1) for the 0·013 in. wire that laminar flow is still persisting up to $R_x = 5 \times 10^5$ and that fully developed turbulent flow does not occur until $R_x = 8 \times 10^5$. The repeat set for this wire, curve (2), shows fully developed turbulent flow at about $R_x = 3 \times 10^5$, transition presumably occurring between there and the wire. The difference between these two sets has been explained by Dutton as arising from the sensitivity of the flow at the leading edge of the plate to small changes in tunnel stream direction. This has probably resulted in the layer approaching the wire being thicker than that predicted by the Blasius theory. We note that, on the basis of the Blasius theory and the estimated R_θ for the 0·013 in. wire, R_θ behind the wire is 201 which is slightly greater than that for amplification to occur which is $R_\theta = 162$. Clearly, we cannot regard this value of R_θ as a lower limit to the R_θ for fully developed turbulent flow, since amplification must occur before transition occurs and this does not always occur suddenly.

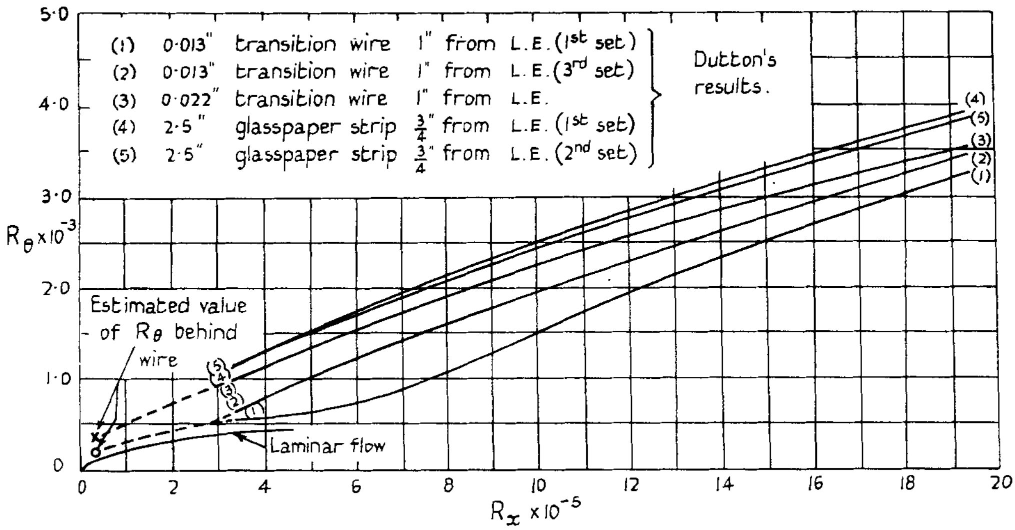

Figure 3. Influence of leading edge conditions on the development of the turbulent boundary layer along a flat plate.

Wire diameter in inches	u_d/U	Ud/ν	ΔR_θ	Total R_θ behind wire
0·013	0·713	423	81	201
0·022	0·952	716	244	365

Table 1. R_θ (in front of wire) = 120, d = wire diameter, u_d = value of u at $y = d$.

In the case of the larger wire (diameter 0·022 in.) the corresponding curve (3) has fully developed turbulent flow characteristics at the first measuring station ($R_x = 3{\cdot}25 \times 10^5$) and it can be extrapolated to the estimated value at the wire without difficulty as shown. This suggests that transition occurs fairly suddenly at the wire where the estimated R_θ is 365. Examination of Dutton's curve suggests that a slightly smaller wire than this would still have had transition at the wire, so that the previously quoted value of $R_\theta = 320$, as the lowest value of R_θ for fully developed turbulent flow, is in agreement with the above assessment of Dutton's results.

The problem of the minimum Reynolds number for fully developed turbulent boundary layers under pressure gradients is more difficult as there appears to be no experimental evidence available. If we take the agreement between the values of R_θ for 'no overlap' of the inner and outer laws and the minimum values of R_θ observed for fully developed turbulent flow in the case of the pipe and plate and being true generally, then it is possible to make some predictions. It is known that the outer limit of the logarithmic region in terms of the boundary layer thickness tends to become less in adverse pressure gradients, so that y_2/δ in equation (10) assumes smaller values than 0·2. Hence in equation (11), the value of $u_\tau \delta/\nu$ for no logarithmic region to exist will increase in adverse pressure gradients. It is also known that θ/δ increases progressively towards separation, as does U/u_τ, and hence the value of $U\theta/\nu$ for 'no overlap' of the inner and outer laws will increase. Hence on the basis of the pipe and plate comparisons it is expected that the minimum value of R_θ for a fully developed turbulent boundary layer will increase in an adverse pressure gradient and decrease in a favourable gradient.

We have noted, from figure 3 of this paper or figure 7 of Dutton's paper, the very considerable influence of the type and magnitude of the transition device on R_θ in the range of $0 < R_x < 2 \times 10^6$ and the same is true of the skin friction given by figure 8 of Dutton's paper. This is of importance, not only in fundamental experiments which are often carried out at low to moderate Reynolds numbers, but also in experiments on ships in towing tanks when the Reynolds numbers are of the order of 10^7–2×10^7. It is also important for compressor blading of gas turbines at high altitudes, when the Reynolds number may be as low as 10^5, and also in experiments in small high speed wind tunnels and low speed atmospheric tunnels, where Reynolds numbers of the order of 10^6 are usual. Another point of importance is that, even when transition occurs at the wire, increase in its size naturally leads to an increase in drag and of momentum thickness downstream, but this increase in drag is not equal to the extra drag of the wire. In fact it will be less because, owing to the increased momentum thickness behind the wire, the skin friction will be reduced downstream. Thus allowance for the drag of the wire in profile drag measurements, for example, by subtracting an estimated wire drag is not correct.

4. Transition devices

On first thoughts it might appear that the primary function of such devices is to produce disturbances which will cause transition from laminar to turbulent flow. On this basis we could never hope to establish turbulent flow nearer to the leading edge than that position at which disturbances in the laminar boundary layer become amplified. In the case of the flat plate this distance is given by $R_x = 6 \times 10^4$, corresponding to $R_\theta = 162$. Clearly any transition device has a drag and as a consequence it increases the momentum thickness of the boundary layer flowing over it. The study of Dutton's results in the previous section shows that the drag aspect is all-important when transition close to the leading edge is desired. It appears from Dutton's results that when $R_\theta < 320$ (the minimum value for a completely turbulent boundary layer) then transition is sudden and close to the wire. If R_θ is less than this behind the wire, laminar flow may persist to much higher values of R_θ and transition take place slowly.

On the other hand, provided $R_\theta < 162$, then a transition device producing vigorous disturbances in the right range of frequencies ought to start the transition almost immediately and this should be complete when $R_\theta = 320$. Also for laminar layers with large R_θ (>320), the production of vigorous disturbances is the important property. Thus any transition device must be considered in terms of both its drag producing and disturbance producing potentialities.

Let us now consider the drag aspect. The drag D per unit span can be related to the increase in momentum thickness $\Delta\theta$ associated with it by

$$D = \rho U^2 \Delta\theta. \tag{16}$$

In this expression we ignore any contribution by the friction in the immediate wake of the device. Now in the case of circular cylinders immersed in thick turbulent boundary layers, Sacks (1956) in an unpublished report found that

$$C_D = D/(\tfrac{1}{2}\rho u_d^2 d) = 0{\cdot}75 \tag{17}$$

over a fairly wide range of $u_\tau d/\nu$. In this relation u_d is the velocity at $y = d$. Hence from equations (16) and (17),

$$\Delta\theta/d = \tfrac{1}{2}C_D(u_d/U)^2,$$

or

$$\frac{U\Delta\theta}{\nu} = \tfrac{1}{2}C_D\left(\frac{u_d}{U}\right)^2\frac{Ud}{\nu},$$

i.e.

$$\Delta R_\theta = \tfrac{1}{2}C_D(u_d/U)^2 R_d. \tag{18}$$

If $(R_\theta)_L$ is the thickness of the laminar layer in front of the wire and if the minimum R_θ for a fully developed turbulent layer is 320, then $(R_\theta)_L + R_\theta = 320$, or

$$R_\theta = 320 - (R_\theta)_L. \tag{19}$$

From equations (18) and (19), the Reynolds number based on the wire diameter is found to be

$$R_d = (2/C_D)(U/u_d)^2\{320 - (R_\theta)_L\}. \tag{20}$$

This equation shows how the size of the transition device is determined from momentum considerations. When $(R_\theta)_L$ is small it is clearly necessary to provide this increment ΔR_θ in momentum thickness by means of the drag. When $(R_\theta)_L > 320$, then no additional increase in momentum thickness is required, but a transition device is still necessary in order to provide disturbances which will cause a rapid transition.

For a wire placed very close to the leading edge so that $(R_\theta)_L = 0$, $(u_d/U) = 1{\cdot}0$ and, taking $C_D = \frac{3}{4}$, $R_d = U_d/\nu = \frac{8}{3}\times 1\times 320 = 850$. For a wire placed as in Dutton's experiments so that $(R_\theta)_L = 120$, we have from laminar boundary layer theory that $u_d/U = 0{\cdot}916$, and hence $R_d = \frac{8}{3} \times (0{\cdot}916)^{-2} \times 200 \doteqdot 636$. A recommended figure, based on experimental evidence for producing transition at a wire which is placed close to the leading edge, is

$$R_d = 600, \tag{21}$$

which is of the same order. This is further support for the idea of a minimum R_θ for fully developed turbulent flow.

When the initial $(R_\theta)_L > 320$, the function of the transition device is to produce disturbances which will bring about transition as close to the wire itself as possible and apart from the rough rule equation (21), it is not possible to give further guidance in the selection of the size of the wire.

Circular wires and other bluff obstacles may, as already mentioned, be expected to have a large drag coefficient and to produce considerable disturbances in the form of cast-off eddies. As regards the drag, Sacks (1956) has found the measured drag of circular cylinders resting on the surface in a boundary layer to be given by $C_D = D/(\frac{1}{2}\rho u_d^2 d) = 0{\cdot}75$, which is about $\frac{2}{3}$ of that found for cylinders in a uniform stream. Recent experiments by Arie & Rouse (1956) on normal plates with a splitter plate placed symmetrically behind give $C_D = 1{\cdot}38 - 1{\cdot}4$, which again is about $\frac{2}{3}$ that of a normal plate in a uniform stream. Incidentally, there is in this paper confirmation of the relation between D and $\Delta\theta$ (equation (16)). On the score of drag, the normal plate is to be preferred to the circular cylinder, since, for a given ΔR_θ, the size of protuberance will be about half that for the circular wire.

As regards the disturbances (when the obstacle is resting on a wall) these do not occur as a regular shedding of eddies in the form of a 'Kármán street'. A separation 'bubble', some 6–8 diameters in length, is formed behind the obstacle. The bubble is composed, in the main, of one large eddy in slow motion, but there is a very disturbed region at the tail of the eddy, which presumably feeds disturbances into the boundary layer.

In the case of the circular wire, when the value of R_θ just behind the wire is just greater than 162 (the value at which disturbances become amplified), it seems that the disturbances are either of the wrong frequency or are not sufficiently large to bring about immediate transition. In Dutton's paper there is a good example of this where the laminar layer proceeds a considerable distance aft of the wire before transition slowly takes place.

5. Choice of transition devices

Circular wires

Circular wires have found favour because of their convenience. They are available in a wide range of sizes and are accurately manufactured. They are fairly easily stuck to the surface, but care is necessary to ensure a uniform spanwise height. From what has been said previously their size is governed both by their ability to thicken the boundary layer and to shed sufficiently strong disturbances.

Normal plate or strip

The use of a thin narrow strip has certain attractions aerodynamically. For a given height, it has roughly twice the drag of the circular wire and presumably the disturbances it produces will also be stronger. Thus for a given increment of ΔR_θ it need be only half the height of the wire. The fixing would give more trouble than for the wire, as would ensuring spanwise uniformity of height. It might be held in a slot in the surface and accurately milled down to size.

Slotted or toothed strip

With this device the drag might possibly be raised above that of the unslotted strip and the teeth would produce strong three-dimensional eddies in the outer part of the boundary layer. This, for fundamental work, may be a disadvantage, as turbulence produced in this region might influence the form of the 'outer' velocity distribution in much the same way as excessive stream turbulence. In the practical problems of suppressing laminar separation by early transition and of delaying turbulent separation by vigorous mixing, they have a definite advantage.

Sandpaper strip

This has been used to produce a rapid thickening of the boundary layer and as a transition device. It has disadvantages for fundamental work in that the results with different strips of the same nominal size are not always capable of being repeated and it is difficult to ensure spanwise uniformity. Also Dutton (1955) has found that the outer velocity distribution does not settle to the same distribution as that which occurs when a wire is used. Presumably, this is because eddies are shed from the tips of the largest spikes and these travel down the outer part of the boundary layer.

Air jets

These have been used to promote transition but they cannot be regarded as a simple device. There are difficulties in ensuring spanwise uniformity and dangers from over-stimulating the outer part of the boundary layer in fundamental work. Distributed blowing through a strip of uniformly porous material could be effective in thickening a boundary layer.

6. Conclusions

In the case of pipe flow there is a lower experimental limit to the Reynolds number for fully developed turbulent flow. From the similarity and close agreement between the curves of c_f *vs* R_θ for both flat plate and circular pipe, it was anticipated that there would be a lower limit to the Reynolds number R_θ for fully developed turbulent boundary layer flow on a plate. Rather limited experimental information confirms this and places the lower limit at $R_\theta = 320$.

There is also a lower limit to the Reynolds number for which there is an overlap of the 'inner' and 'outer' laws, the logarithmic region. Estimates of the Reynolds number agree fairly closely with the limiting

Reynolds for turbulent flow in the case of the pipe and flat plate. If this agreement is to be expected generally, then, on this basis of 'no overlap', it would seem that the lower limit of R_θ for fully developed turbulent flow should decrease in favourable pressure gradients and increase in adverse gradients.

This concept of a minimum Reynolds number for turbulent flow is important when considering the drag of flat plates at medium Reynolds numbers and in considering the size of transition device to be used. For quick transition near the leading edge, the drag of the device and its ability to produce disturbances are both important. An approximate theory has been developed on the basis of a minimum R_θ for turbulent flow which, in the case of circular wires, leads to a 'wire' Reynolds number in fair agreement with that suggested by experience.

As regards the choice of transition devices, the conclusion is that a normal strip has the least height to achieve transition, but the circular wire is more convenient. When suppression of laminar separation and delay of turbulent separation is important, it is thought that a toothed spoiler or slotted strip should give good results; but the generation of eddies in the outer part of the boundary layer is objectionable in fundamental work, as the 'outer' part of the velocity distribution may be affected.

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